Corrigendum to "Marginal Effects in Interaction Models: Determining and Controlling the False Positive Rate"*

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After publication of Esarey and Sumner (2018), we recognized substantively significant errors in the paper. We correct those errors here. We discovered these errors as part of work on another, unrelated paper to which one co-author hoped to apply similar methods. To summarize, the section titled "Underconfidence is possible for conjoint tests of theoretical predictions" and the subsections "Underconfidence corrections for estimated marginal effects" and "Suggestion 3: specify theories with multiple predictions in advance and used bootstrapped critical t statistics to maximize empirical power," including Tables 3 and 5, are incorrect. The related "Prediction-corrected 90% Confidence Interval" in Figure 2 is based on this erroneous procedure and should be ignored. Contrary to Esarey and Sumner (2018), the procedure for generating $(1 - \alpha)$ confidence intervals described in Brambor, Clark and Golder (2006) *can* be used to conduct a test with size at most α where multiple hypotheses are being jointly tested and all predictions must be jointly confirmed in order to accept the theory (Silvapulle and Sen 2005, Section 5.3; Casella and Berger 2002, Section 8.2.3 and 8.3.3). The proposed bootstrapping procedure does not do so, and should not be used.

The portions of Esarey and Sumner (2018) related to overconfidence of confidence inter-

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vals described in Brambor, Clark and Golder (2006) in situations where individual marginal effects are being tested and reported, and the related FDR and FWER-controlling corrections, are (to our knowledge) accurate. We do think it important to add that the Benjamini and Hochberg (1995) procedure is only formally proved to control the false discovery rate under independence of test statistics or positive regression dependency in the subset of true null hypotheses (Benjamini and Yekutieli, 2001); however, our own simulation evidence (see also Reiner-Benaim, 2007) appears to indicate that the procedure works adequately in the regression interaction context. We also think it important to add that the properties of confidence intervals constructed using the Benjamini and Hochberg (1995) procedure are discussed in Benjamini and Yekutieli (2005, p. 72), and in particular that these confidence intervals are formally proved to control the false coverage rate (i.e., "the expected proportion of parameters not covered by their [confidence intervals] among the selected parameters [statistically significant effects under the Benjamini and Hochberg (1995) procedure]"), not necessarily the overall false coverage probability for all parameters (i.e., those where the null hypothesis is not rejected), when the test statistics are independent (see also Benjamini, 2010).

Details

First, the section of the paper entitled "Underconfidence is Possible for Conjoint Tests of Theoretical Predictions" is incorrect. Contrary to our argument, the Brambor, Clark and Golder (2006) method produces confidence intervals associated with accurate significance tests when *jointly* testing multiple hypotheses (although not when *separately* testing multiple hypotheses). That is, in situations where a previously specified theory makes multiple predictions for $(\partial y/\partial x|z)$ at different values of z for a linear model:

$$y = \beta_0 + \beta_x x + \beta_z z + \beta_{xz} xz \tag{1}$$

then testing these hypotheses using $(1-\alpha)$ confidence intervals generated using the Brambor, Clark and Golder (2006) method will jointly reject all the null hypotheses at most α proportion of the time. For example, if $z \in \{0, 1\}$, and a researcher pre-specifies alternative hypotheses that $(\partial y/\partial x|z = 0) < 0$ and $(\partial y/\partial x|z = 1) > 0$, separate t-tests rejecting each null separately using t-tests with size α will jointly reject both nulls at most α proportion of the time. This is discussed and proved in Silvapulle and Sen (2005, Section 5.3), especially in proposition 5.3.1, who in turn cite (inter alia) Lehmann (1952); Berger (1982); Cohen, Gatsonis and Marden (1983); and Berger (1997). It is also discussed in Casella and Berger (2002, Section 8.2.3 and 8.3.3).

At the start of this section, Esarey and Sumner (2018) incorrectly calculates the probability of a false positive result for jointly testing directional predictions. Let $(\partial y/\partial x|z = z_0)$ be abbreviated as $ME_x^{z_0}$. Esarey and Sumner (2018, p. 1157) states that:

$$\begin{aligned}
& \sup \Pr(\text{false positive}|ME_x^0 \leq 0 \vee ME_x^1 \geq 0) \\
= \ & \Pr\left[\left(\widehat{ME}_x^0 \text{ is stat. sig. and } > 0|ME_x^0 = 0\right) \wedge \left(\widehat{ME}_x^1 \text{ is stat. sig. and } < 0|ME_x^1 = 0\right)\right] \\
= \ & \Pr\left(\widehat{ME}_x^0 \text{ is stat. sig. and } > 0|ME_x^0 = 0\right) * \Pr\left(\widehat{ME}_x^1 \text{ is stat. sig. and } < 0|ME_x^1 = 0\right) \\
= \ & \alpha^2 = 0.05^2 = 0.0025
\end{aligned}$$

but this calculation would only be true if $ME_x^0 = 0$ and $ME_x^1 = 0$. The null hypothesis space stated includes the possibility (for example) that ME_x^0 is large but $ME_x^1 = 0$. Consequently, $\sup \Pr(\text{false positive}|ME_x^0 \leq 0 \vee ME_x^1 \geq 0)$ can be as high as the α value of any of the individual tests (in this case, 0.05), as stated in Silvapulle and Sen (2005) and Casella and

Berger (2002). All the quantities in Table 3 of Esarey and Sumner (2018) are based on similar miscalculations.

Second, the subsection of Esarey and Sumner (2018) titled "Underconfidence corrections for estimated marginal effects" and related material in the subsection titled "Specify theories with multiple predictions in advance and use bootstrapped critical-t statistics to maximize empirical power" is incorrect. In particular, the bootstrapped critical-t statistics reported in Table 5 of Esarey and Sumner (2018) should not be used. They correspond (for non-zero predictions) to t-values that occur 5% of the time or less in the situation where all marginal effects are equal to zero; Table 5 in this paper confirms that the procedure works, but only for a point null hypothesis for which all marginal effects are simultaneously equal to zero. We have removed the corresponding function for calculating these t-statistics from our `R` package (while leaving in the function to calculate a t-statistic corresponding to a given false discovery rate).

Finally, in our reanalysis of Clark and Golder (2006), the appropriate confidence intervals in Figure 2 for a joint test of the proposed hypotheses is the original 90% confidence interval reported by Clark and Golder (2006). The "prediction-corrected" 90% confidence interval should be ignored.

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