Causality and statistical dependence

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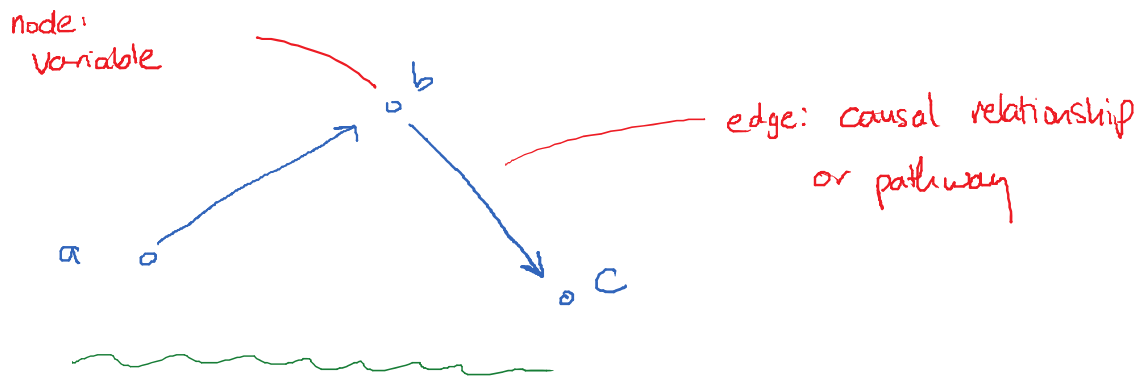
What is a Bayes net? A way of recovering information about the structure of statistical dependencies among variables.



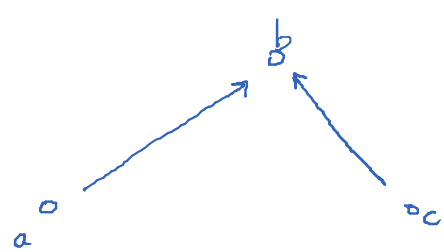
a causes b, b causes c

Causality: if a causes b, then changes in a will create consistent changes in b holding all other factors constant.

a causes b $\xrightarrow{\text{implies}}$ $f(a,b) = f(b|a) \cdot f(a)$

b causes c $\longrightarrow$ $f(b,c) = f(c|b) \cdot f(b)$

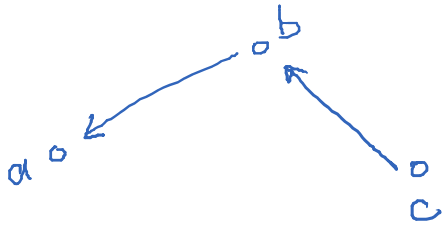
$$f(a,b,c) = f(c|b) \cdot f(b|a) \cdot f(a).$$



$$f(a,b,c) = f(a)\, f(c)\, f(b|a,c)$$

Bayes nets examine a pattern of statistical dependencies in a data set and attempt to come to a conclusion about the

structural network of causal relationships that produced those dependencies.



$$f(a,b,c) = f(a|b)\, f(b|c)\, f(c)$$

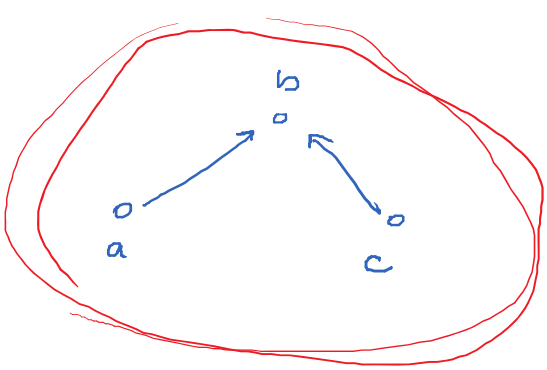
use Bayes' rule : $f(a|b) = \dfrac{f(b|a)\, f(a)}{f(b)}$

$$\frac{f(b|a)f(a)}{\cancel{f(b)}} \cdot \frac{f(c|b)\cancel{f(b)}}{\cancel{f(c)}} \cdot \cancel{f(c)}$$

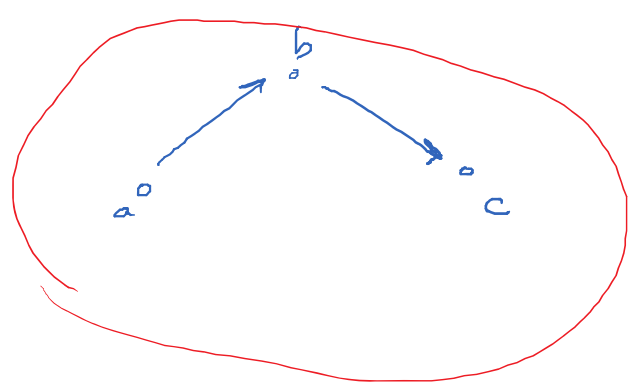
$$f(b|a) \cdot f(a) \cdot f(c|b$$

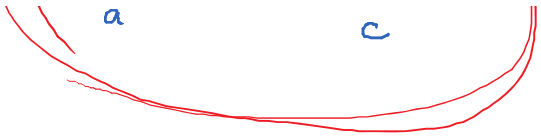
All graphs imply a unique pattern of statistical dependencies, but not all patterns of statistical dependency uniquely imply a graph.

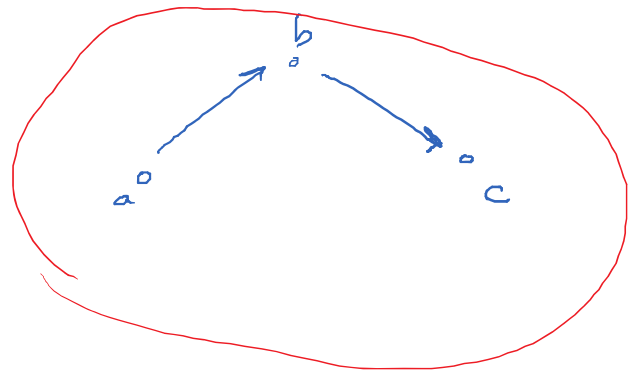
This interferes with a researcher's ability to infer networks from patterns of dependency.

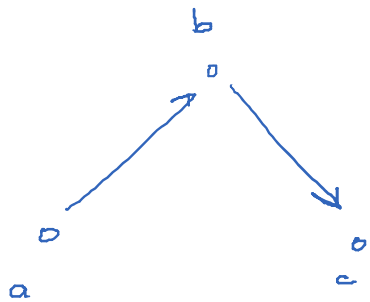


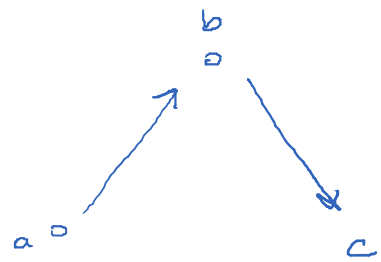
$$f(a)\, f(c)\, f(b|a,c)$$

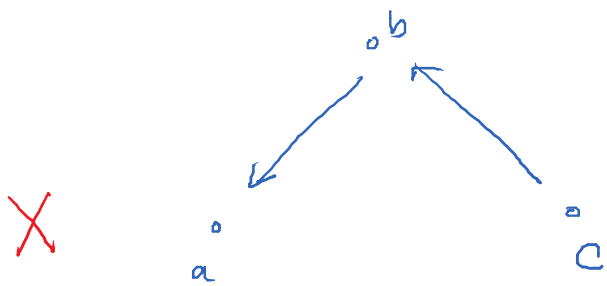








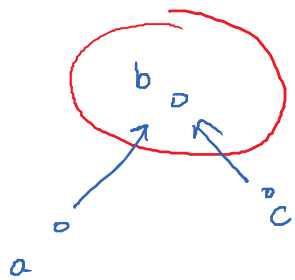


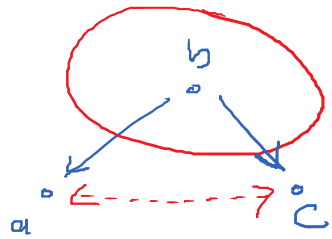


$$f(a|b)\,f(b) \Longleftrightarrow f(a)\,f(b|a)$$

$$\frac{f(b|a)\,f(a)}{\cancel{f(b)}}\,\cancel{f(b)}$$

$$f(b|a)\,f(a).$$





$$f(a,c) = f(a)\,f(c)$$

$$f(a,c) \neq f(a)\,f(c)$$

$$f(a,c|b) = \frac{f(a,b,c)}{f(b)}$$

$$= \frac{f(b|a,c)\cdot f(a)\cdot f(c)}{f(b)}$$

$$\neq f(a|b)\cdot f(c|b)$$

$$f(a,c|b) = \frac{f(a,b,c)}{f(b)}$$

$$= \frac{f(a|b)\,f(c|b)\cdot \cancel{f(b)}}{\cancel{f(b)}}$$

$$= f(a|b)\,f(c|b)$$

D-separation

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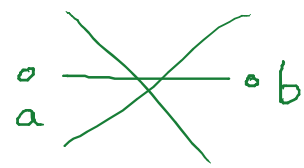
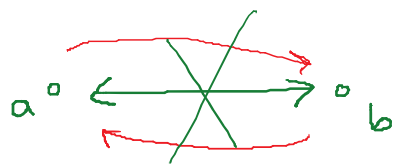
Judea Pearl : d-separation

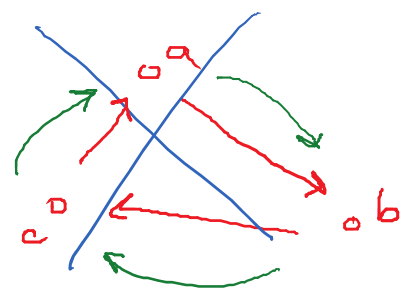
applies to directed acyclic graph

directed : graph in which edges are directional.



acyclic : graph has no loops; there is no pathway from any node back to itself.





Consider two sets of variables, A and B, and a third set of conditioning variables, C.

Pearl demonstrates: $f(a,b \mid c) = f(a \mid c)\, f(b \mid c)$ for all $a \in A$, $b \in B$ if, for every path between a and b:

a and b:

d-separation

either

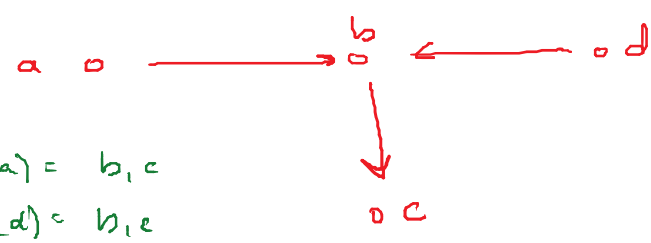
$\boxed{1}$ the arrows on each path meet head to tail, or tail to tail, at a node in C,

or

$\boxed{2}$ the arrows meet head to head at a node not in C and whose descendants are not in C.

any node connected tail to head from the target

"ancestors": reverse descendancy (any node connected head to tail to target)



$d(a) = b, c$

$d(d) = b, c$

$a(c) = b, a, d$

$a(b) = a, d$

Pearl's idea opens the door to ways to systematically

construct the network(s) implied by a data set.

Mapping dependencies that result from causal connections

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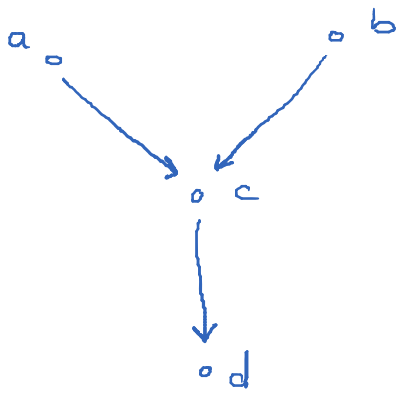

Im(a ~ b) → no relationship.

Im(a ~ b + c) → b related ✓
c related. ✓

Im(a ~ d) → d related ✓

Im(a ~ d + c) → c related
d not.

Im(a ~ b + c + d) → b related
c related
d not.

Structuring regression analysis around a causal mapping

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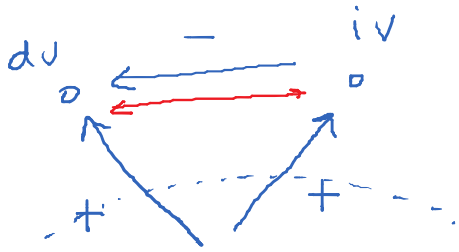
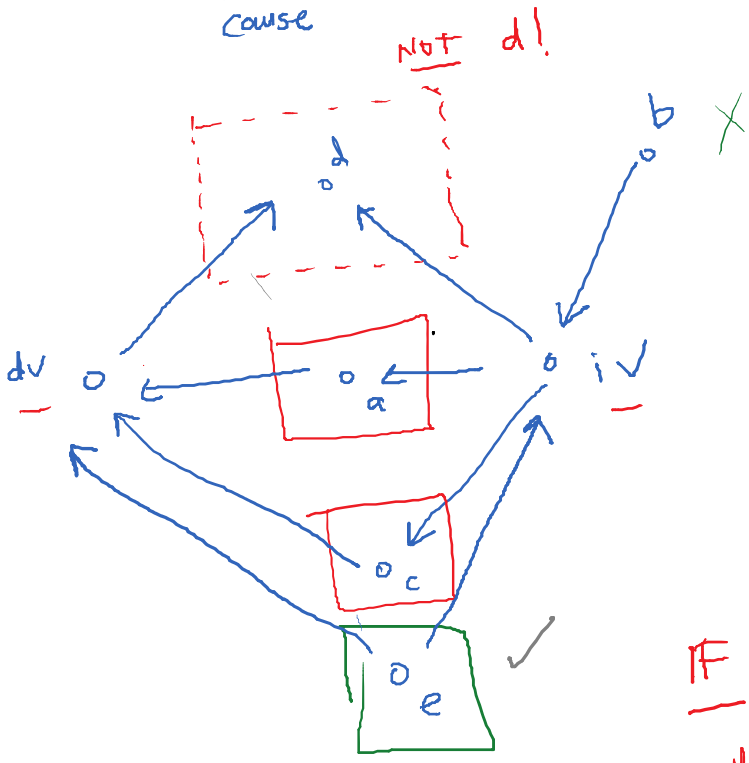


"back door"

spurious cause



What should we control for?

IF we only want direct paths from iv to dv: control for a & c.

IF we want to include indirect pathways

iv → X → dv

then don't control for a & c.

Inference in a Bayes net

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